Waste Not, Want Not: Putting Recyclables in Their Place

Team 1770

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1 Summary

Plastic has become ubiquitous, even omnipresent, in modern society; not since the Stone Age has a material been used so much. The list of materials which contains plastic seems never-ending - it is in jewelry, body armor, automobiles, computers, cups, furniture, and even clothing. The advantage of plastic over its contemporaries - wood, metal, stone, cloth - comes from its versatility and cheapness; it can be as hard as a diamond, or as fragile as a spiders web to suit the needs of the consumer, but for much less than any other material. However, the situation is not nearly so rosy as it would appear on the surface. Every item, no matter how beneficial, will eventually run its course. All that is left to do with it is to dispose of it - but where? Plastics very versatility, which proved so useful in life, now comes back as a haunting specter that will not disappear. It will not biodegrade, and mankinds answer has been to shut it away in the ground. Obviously, such a situation is intolerable.

Our firm was initially tasked with projecting the amount of plastic which would arrive, one way or another, in landfills over the course of time, specifically ten years. To this end, we created a spatiotemporal compartmental model that mathematically represented the flow of plastic from its unprocessed resin form to the manufacturing process, to the market, and then to either the recycling facility or the landfill. This model was segmented into seven parts according to the seven types of plastic classified by the Plastics Industry Trade Association, allowing a future user of this model to quantify the different impacts of increased recycling of any one type of plastic. This model yielded a 17.8% increase in landfill contents due solely to plastic waste over the next 10 years, indicating a dire need for increased plastic recycling in the United States.

We then modelled three different methods for collecting recyclable materials: single-stream collection without tax incentives, single-stream collection with tax incentives, and presorted drop-off recycling. All three were modelled for the efficiency of their collecting service, in terms of weight of recyclable material gained per dollars spent. Factors considered in constructing these models were the amount of recyclable mass each household would be willing to recycle, the convenience of recycling, the initial investment required to enable a recycling system, the likely life-expectancy of the equipment, the fuel expenditure of the vehicles, and the amount of times each vehicle would visit a specific household. For the second and third collecting systems, respectively, the change in convenience was modelled, as were changes in start up costs.

The models were subsequently applied to three cities: Fargo, North Dakota; Price, Utah; and Wichita, Kansas. Unsurprisingly, all three models produced different efficiency ratings in different cities, but all of the cities were better off with the recycling plans put into place. These results give us reason to believe that the Environmental Protection Agency (EPA), can and should put into place nationwide standards for the collection of recyclable materials. The single-stream collection method with tax incentives should be put into place because of all of the models we considered, it returned the best results.

That said, the single-stream and tax incentive method should not be the only method of increasing plastic recycling in the United States. We also provide a number of other suggestions for extending recycling guidelines to the national scale, in the hopes that one day the United States will be an environmentally sustainable society.

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2 Introduction

2.1 Background

Wallace Carother's invention of nylon in 1934 while working for DuPont Industries created a commercial juggernaut; within a matter of years, nylon was used in fields ranging from toothbrushes to parachutes (6). What's more, he launched a revolution, for over the next half-century, plastic would supplant traditional materials. Wood, stone, leather, metal, and even glass have been displaced to an extent by the inexorable march of all forms of plastic; plastic has achieved such a dominating position because of its versatility and cheap cost (3).

Unfortunately, nothing lasts forever. Plastic goods, like all others, are eventually no longer needed by their owners, and are disposed of; the majority is thrown out, while the remainder is recycled. In 2010, more than thirty-one million tons of plastic waste was generated (13). The amount of waste in landfills is ever-growing, and will remain lurking beneath the ground for hundreds of years.

2.2 Restatement of the Problem

The United States Environmental Protection Agency (EPA) has asked our firm to:

1. Create a model which estimates the amount of plastic in landfills over time, and predict the amount of waste in 10 years.
2. Develop a model, independent of current recycling processes, which a city can use to determine which recycling method it should adopt.
3. Develop a mathematical model, independent of current recycling processes, that will take in city data in order to recommend the most effective recycling method.
4. Demonstrate the model works by applying it to Fargo, North Dakota; Price, Utah; and Wichita, Kansas.
5. Use the model to reconsider the feasibility of regulating and/or mandating recycling for all towns and states, and make recommendations to the EPA based on the feasibility.

2.3 Global Assumptions

1. The sum of the plastics in each of the 7 plastic classifications accounts for the total amount of thermoplastic in the market.
2. We will assume that the market is in equilibrium, such that demand is equal to supply. In theory, any market will tend towards equilibrium over a long enough period of time. In addition, the United States has not recently encountered a dramatic shift in supply or demand.
3. There will be no political oppositions to such proposals. It is assumed that it is such a decisive issue that the majority of America will support the cause.
4. There will be no major disasters, wars, or economic disasters during the time period which the model considers.
5. All the data obtained from the sources is accurate.

3 Analysis

3.1 Model 1: Amount of Plastic Entering Domestic Landfills

3.1.1 Rationale

The flow of plastic typically starts at the polymerization stage, during which raw materials such as crude oil and cellulose are refined and used to synthesize resins through addition or condensation reactions. Subsequently, additives such as glass/carbon fibers, antioxidants, and plasticizers are used to reinforce, prevent the degradation of, and increase the flexibility of the plastic. The plastic product is then completed through a finishing process such as extrusion, injection molding, or compression molding.

Due to its low-cost production and great versatility, plastic is widely used in manufacturing. In fact, the plastic industry is the third largest in the United States, employing over 885,000 members of the labor force and producing over $380 billion in annual shipments (39). Unfortunately, the high production levels of plastic is clearly unsustainable, as crude oil supply is rapidly dwindling in regions across the globe (26). Furthermore, plastic also takes a long time to degrade, often accumulating in landfills and polluting the nearby ground, air, and water. Among the numerous detrimental effects of plastic waste accumulation include human/animal ingestion of harmful plastic additives, transportation of invasive species by plastic waste into regions where they are non-endemic, and pollution of groundwater. Thus, the objective of this model is to predict the amount of plastic waste entering landfills over the next ten years.

To achieve the aforementioned goal, we propose a compartmental model that represents the flow of plastic through the manufacturing process and the market until it reaches a landfill. As plastic is processed, manufactured, sold, and disposed, it moves from compartment to compartment via inflows/outflows with unique transition rates. We use these transition rates to derive a system of ordinary differential equations relating the change in the amount of plastic within each compartment to the passage of time. As a baseline, the recycling rate and other relevant transition rates from 2010 are used as inputs for the model. We then solve the system of ODEs to obtain analytical functions of time for forecasting the amount of plastic entering landfills over the next 10 years.

3.1.2 Assumptions

1. There is not a significant amount of thermoset material in production in the plastic industry. The analysis, therefore, considers solely the thermoplastic materials in the market. The thermosets are insignificant, for they comprise less than 5% of the main plastic industries. (14).
2. All compartments in the given model are mutually exclusive. A unit of plastic can be assigned to only one compartment at any given point in time.
3. The amount of plastic production that can be attributed to net exports (exports − imports), 4%, is negligible (37).

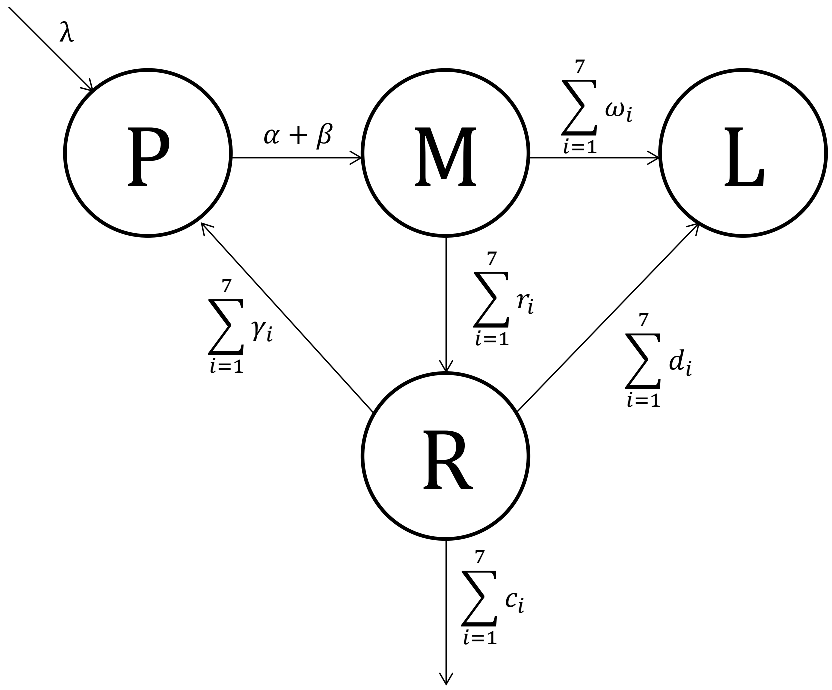

Figure 1: The flowchart of the compartmental model.

3.1.3 Model

Unrefined resin products enter the system at rate λ and enter P, the production compartment, where they are processed and prepared for manufacturing. Generally, almost all resin products are successfully manufactured and proceed at rate $\alpha + \beta$ to compartment M, the market. α is the rate at which packaging products enter the market, whereas β is the rate at which non-packaging products enter the market. Here we assume that all plastic products progress either to recovery, compartment R, or landfills, compartment L. If a plastic unit progresses to compartment R, then it can be recycled and returned to production, compartment P, at rate γ, or it can be combusted at rate c. After combustion, the plastic is released into the environment, no longer circulating in the system, and thus it is represented as an outflow out of the system. However, the recycling process is not fully efficient, and the recycling compartment experiences a waning rate, d_i, at which plastic products move from compartment R to compartment L, the landfill. Furthermore, plastic products on the market that do not move to compartment R must eventually move to the landfill, compartment L.

Another factor to consider is the varying flow rate of each of the different types of plastics. In 1988 the Plastics Industry Trade Association implemented a Resin Identification Code (RIC) system for classifying plastic products into 7 different classes, identifying plastic containers with a stamped number to designate their plastic type (39). The products in each class are recycled, reused, and disposed at different rates. As such, the model is run seven times, in each iteration considering a different class of plastic for the parameters.

The movement of compartment materials is depicted in the system flowchart shown in Figure 1. In addition, the names and definitions of all parameters of the model are listed in Table 1. Using the rates of inflow and outflow between compartments as designated by

Variable or Parameter	**Definition**
P	Compartment of plastic production
M	Compartment of plastic sold on the market
L	Compartment of landfill
R	Compartment of recycled plastic
λ	Inflow rate of plastic
α	Production rate of plastic packaging products
β	Production rate of non-packaging plastic products
ω_i	Waste rate of plastic products in class i
r_i	Recycling rate of plastic products in class i
c_i	Combustion rate of plastic products
γ	Reuse rate of recycled plastic
d	Waning rate of plastic recycling (inefficiency)

Table 1: Definitions of model variables and parameters. All compartments are expressed in units of thousand tons of plastic, and all parameters are expressed as proportions of change per time (yr^{-1}).

the model flowchart, we derive four first-order differential equations for the rates of change of compartments P, M, R, and L, with respect to time t in years after 2013. Each of these equations is composed of "inflow" and "outflow" terms representing the rate of flow into and out of each compartment.

$$\frac{dP}{dt} = \lambda + \sum_{i=1}^{7}(\gamma_i(t) \cdot R) \tag{1}$$

$$\frac{dM}{dt} = (\alpha(t) + \beta(t)) \cdot P(t) - \sum_{i=1}^{2}((r_i(t) + \omega_i(t)) \cdot M(t)) \tag{2}$$

$$\frac{dR}{dt} = \sum_{i=1}^{2}(r_i(t) \cdot M(t)) - \sum_{i=1}^{2}((\gamma_i(t) + d_i(t) + c_i(t)) \cdot R(t)) \tag{3}$$

$$\frac{dL}{dt} = \sum_{i=1}^{2}(\omega_i(t) \cdot M(t) + d_i(t) \cdot R(t)) \tag{4}$$

Since the values of the rates of flow between compartments are not constant, their values depend upon coefficients defined as functions of time. Data sourced from the EPA (37) and the American Chemistry Council (7) was analyzed for the past ten years to determine past and future trends. There was a strong correlation between the year and the values of $p_1(t)$ and $p_2(t)$, with R^2 values for a linear fit of approximately 0.98. Correlations between the year and other variables were also determined, as shown in graphs

$$P(t) = 5.85987 \times 10^{41} \cdot e^{-4.27233\times10^{-2} \cdot t} \tag{5}$$

$$p_1(t) = \alpha(t) = (0.0104164 \times t - 20.4491) \tag{6}$$

$$p_2(t) = \beta(t) = (-0.0104164 \times t + 21.4491) \tag{7}$$

$$r_1(t) = 0.00722593 \times t - 14.3832 \tag{8}$$

$$r_2(t) = 0.00124816 \times t - 2.47655 \tag{9}$$

Certain other coefficients were determined to remain relatively constant in the short term. In particular, $c_1(t) = c_2(t) = 7.7\%$, $d_1(t) = d_2(t) = 11\%$ (36), and thus $\gamma_1(t) = \gamma_2(t) = 100\% - (c_1(t) + d_1(t)) = 81.3\%$.

3.1.4 Results

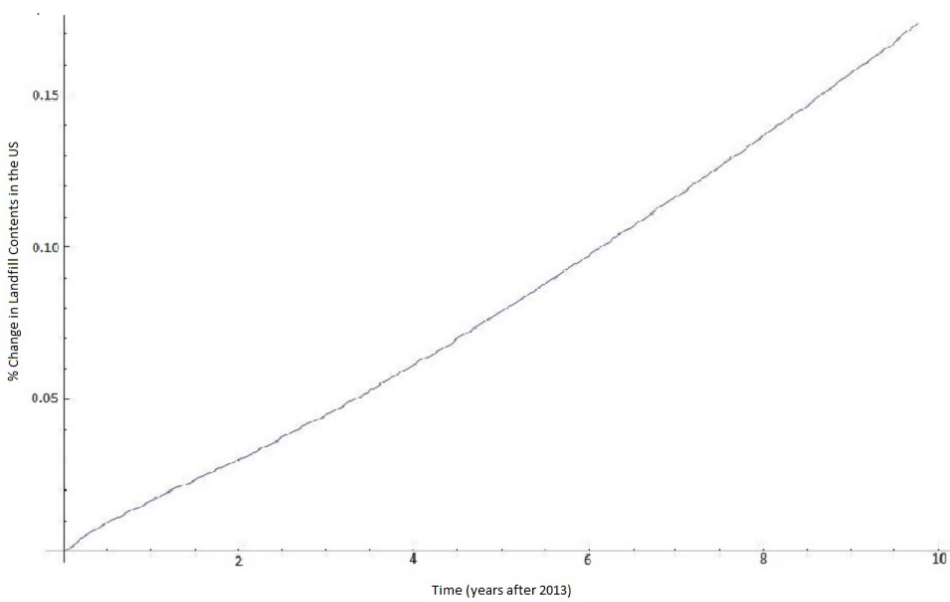

Figure 2: Graph of the percent change in the contents of the landfill compartment.

We then solve this system of differential equations using the computer algebra system Mathematica, with initial conditions set to $P(0) = 50,000, M(0) = 44,704, R(0) = 1,240,000$, and $L(0) = 31,000,000$. These initial values were obtained from the literature. (Note that since we only care about the change in landfill contents, and the landfill compartment is purely a sink, the initial value of $L(t)$ is of no importance.)

As shown in Figure 2, the percent change in landfill contents (due only to plastic waste) of the United States over the next 10 years will increase at an almost linear rate, reaching a 17.8159% increase in 2013. This is reasonable because plastic waste trends have shown similar linear growth in the last 10 years.

3.2 Modeling the Optimal Recycling Method

In order to determine the most effective method of collecting recyclables, we consider three separate models: single-stream curbside pickup, single-stream curbside pickup with government tax incentives, and drop-off locations where presorted recyclables could be deposited. We decided to define efficiency both in terms of weight of recyclables collected per dollars spent - as it would be applicable for budgets of every size - and in terms of recyclables collected per total amount which could be collected - as it allows for a more uniform measurement.

3.2.1 Single-Stream Rationale

The recycling process varies from location to location. Typically, the process involves the disposal of recyclables by the consumers, later picked up by dedicated garbage disposal trucks, which are then brought to specified processing centers, and are finally recycled back into the industries for use. Though seemingly effective, the recycling process fails to attract 30% of the national population and, of the 70% who do recycle, only 34.1% of the possible recyclables end being recycled. As a result, it became apparent that a multitude of recycling programs had to be investigated in order to determine which proposes the best solution, leading to the greatest efficiency in the recycling program.

Subsequently, the model had to be determined and formed without the use of current recycling data and further sought to quantify the efficiency in Single-Stream Recycling (without tax incentives). In essence, this recycling technique involves the customer disposing of all their recyclable waste into a single basket, without discrimination as to the plastic type, form, or other materials. As a result, the customers would not require as much effort in order to recycle should they choose to adopt this program, as they would no longer need to separate the plastics or other materials. To develop the model, we determined each of the related factors with the efficiency and how each would impact the overall efficiency; after obtaining empirical data for each of the variables, the constant was solved for, which was then used to determine the efficiencies for each of the other cities under investigation.

3.2.2 Single-Stream Model

Within the last generation, the single-stream collection method has overtaken the archaic dual-stream method as the prevalent one in America (5). Instead of having multiple compartments in a waste disposal vehicle, as mandated by the dual-stream system, the single-stream system has one compartment for recyclables, greatly increasing the efficiency (5). In switching to a single-stream model, we will consider the use of the TW1120199SL Side Loader Garbage Truck from Autocar and a 96 gallon waste disposal bin (1).

The efficiency is defined in terms of weight of collected recyclables per dollar is written as:

$$E(x) = \frac{M}{P(x)} \tag{10}$$

Where $E(x)$ measures the efficiency, M the total mass of the materials recycled, and $P(x)$ is the money spent for the recycling program. The total expected mass of the collected recyclables, M, is expressed as:

$$M = N(\frac{r}{w} \times m \times K \times L) \tag{11}$$

Where r is the rate of recycling, w is distance, m is total weight of material that can be recycle, N is the number of times a waste disposal vehicle goes by, L is the number of houses a waste disposal vehicle can visit, and K is a constant. The rate of recycling for a set population, and the mass of the materials they can recycle are both directly related to the total mass recycled; as they increase, the total mass of recycled materials increases. It has been shown that the current recycling rate, as it exists now, continues at a more or less continuous level in the future, and is unlikely to change (4); thus, this value is kept constant. There is a convenience factor taking into account the distance a person has to walk to bring their trash to the curb, as the lesser the distance one has to walk to the curb, the more convenient it becomes for the resident. In addition, as L is the number of houses a waste disposal vehicle can visit before it needs to return to the wast disposal center, and N is the number of times a waste disposal vehicle has collected its cargo, they together give the amount a single vehicle could be expected to collect. K is a constant factor which transforms the units of the equation into the units of mass, and scales it appropriately. Altogether, this represents the weight of the recyclables taken in by N number of waste disposal vehicle visits. The cost of running the processing center was not factored in because in terms of pure efficiency, it is constant across the majority of the systems we will be analyzing.

The cost efficiency, however, is determined by more than simply the total mass of recyclables collected. It also depends on the cost of collecting the recyclables, given by:

$$cost = N(2D + L \times d) \times f \times p + \frac{C}{7} + W \times 7 + L \times c \tag{12}$$

Where D is the distance from the waste disposal vehicle to the processing center, d is the average distance between houses, f is the fuel efficiency of the waste disposal vehicle, p is the price of gasoline, C is the cost of a waste disposal vehicle, W is the salary of the waste disposal worker, and c is the cost of the 96 gallon disposal bin. The term $N(2D+L\times d)\times f\times p$ gives the fuel cost for the truck to complete its journey by multiplying the total distance traveled $(2D + L \times d)$ by the fuel efficiency and cost of gasoline and the number of runs the truck has to make. W is multiplied by the life of the vehicle to represent the total cost of operating the truck. The other two terms represent starting costs; $L \times c \times 63$ is the cost of the number of waste disposal bins needed for one run, and $C/7$ is the cost of the waste disposal vehicle divided by its average lifetime, 7 years (12).

The price efficiency, therefore, is given by:

$$E = \frac{N(\frac{r}{w} \times m \times K \times L)}{N(2D + L \times d) \times f \times p + \frac{C}{7} + W \times 7 + L \times c} \tag{13}$$

The volume of the waste disposal vehicles is 30 cubic yards (1) after compaction; before compaction, the volume of the waste which would fit in the vehicle is about 50 cubic yards (2). As the volume of the garbage disposal bins utilized by our system is 96 gallons, if all of the bins are filled completely, it would take 105 bins to fill the truck to maximum capacity.

The trucks will only stop at 105 houses; any more, and there is a chance, however small, that it could overflow the waste disposal vehicle; L equals 105.

The cost of the vehicle, C, is equal to \$225,900, and the fuel efficiency, f is 6 miles per gallon (1). There is only one workingman per vehicle, and his or her salary (W) is \$39,830 (11). c, the cost per garbage disposal bin, is \$89.98 (10), and the price of gas per gallon, p, can be found for the local, or national, community for which the model is being applied.

The value of the constant, K, can be found by using recent historical data, and the equation:

$$M = N(\frac{r}{w} * m * K * L) \tag{14}$$

The Township of South Orange, New Jersey, was used to determine the K value; it has recently switched over to the single-stream system, and is likely to serve as an excellent model for towns which would be forced to switch. It has had 14 recycling collections since it has switched over - leaving it with an N value of 14 (20). L, as always, is 105 visits, for South Orange uses the very trucks we are recommending and w for South Orange is 76.96 feet (18). The recycling rate should be one hundred percent, as recycling was recently made mandatory by the law, but the crime rate of 3% brings it down to 97% (19). m is the total weight which could have been recycled; it can be found by multiplying the consumption of recyclable materials per capita per day by both the population and time constraints present in South Orange's experiment. The average person consumes 3.8 pounds of recyclable materials per day (18), it has been 235 days since South Orange began their program (20), and the population of South Orange is 16,207 people (20).

$$m = 3.8 \text{ pounds/day} \cdot \text{person} \times 235 \text{ days} \times 16207 \text{ people} \times \frac{1 \text{ ton}}{2000 \text{ pounds}} \tag{15}$$

$$= 7236 \text{ tons} \tag{16}$$

As the total amount of recyclable goods collected in South Orange was 898 tons (20), the value of K can be solved for by manipulating the equation.

$$K = \frac{898\text{tons}}{\frac{0.97}{79.96\text{text}feet} * 7,236\text{tons} * 105 * 14} = 0.0066978 \tag{17}$$

The model for the single-stream recycling method is:

$$E = \frac{N(\frac{r}{w} * m * 0.0066978 * 105)}{N(2D + L * d) * 6\frac{miles}{gallon} * p + \frac{\$225,900}{7} + \$39,830 * 7 + \$9447.9} \tag{18}$$

3.2.3 Results of the First Model

From our first model, it is very clear that the efficiency of many towns in the United States, with respect to the amount of recyclables and the amount of cash put towards efforts, is drastically low. It is immediately clear that only increasing the amounts put towards recycling programs will not cause a major change in tons of recyclables. Rather, from the factors isolated in our model, in order to cause a significant increase in the amount of recyclables, we must increase the efficiency of our disposal vehicles, increase percentage of the population that recycles, and increase convenience. One of the major losses in efficiency was due to fuel inefficiency of the vehicles that pick up waste. Waste Management, one of the largest private waste contractors, has recently upgraded their fleets towards hybrid vehicles to improve vehicle performance. In addition, the major selling point for the single-stream model was the increase in convenience for the consumers. Although there is a limit to how convenient a city can make recycling, it is still a factor to consider. Finally, in some cities, a major loss in efficiency was due the fact that only a small percentage of the population participated in recycling efforts. Thus, public outreach and other efforts to inform the public are necessary.

3.2.4 Rationale of the Second Model

The second model, namely the Single-Stream Recycling (without tax incentives), sought to accomplish the same task as did the first; however, apparent from the name, this model differed from the first in that it considers tax incentives in place advocating the adoption of recycling programs. Though many cities consider adopting a policy of punishing people who excessive use of the traditional waste disposal system as opposed to recycling, through both social and behavioral studies, it has been determined that positive reinforcement is stronger than negative reinforcement (23). In other words, adopting a policy that rewards those who recycle would lead to greater increases in the efficiency contrasted with one which punishes those who do not recycle.

As a result, the model was aimed at determining whether increasing the tax incentives a specified amount would yield higher recycling rates. This model, as with the first, did not include the current recycling rate and did not discriminate between various recyclable materials, for it is still a single-stream recycling program. However, after evaluating for the efficiencies for each of the cities, they were compared with the original efficiencies to determine whether they would yield significant percent changes, as Price already implements a tax incentive program.

3.2.5 Single-Stream Recycling with Tax Incentives

In order to further increase the percentage of the population that participates in current recycling efforts, some cities have attempted to add a fixed price for every container of garbage used to encourage recycling through negative reinforcement. However, instead of negative reinforcement, we have decided to focus on the effects of positive reinforcement. The work of many psychologists have showed that positive reinforcement was much more efficient than negative reinforcement (23). Thus, instead of placing a price on each can of garbage, our group has looked at the effects of tax incentive. Because this method still utilizes single-stream recycling, the equation from the previous model still applies.

$$E = \frac{N(\frac{r}{w} \times m \times K \times L)}{N(2D + L \times d) \times f \times p + C/7 + L \times c} \tag{19}$$

However, modifications must be made in order to account for the effects of the tax incentive as such.

$$E = \frac{N(\frac{r \times t}{w} \times m \times K \times L)}{N(2D + L \times d) \times f \times p + C/7 + L \times c} \tag{20}$$

An extra factor, t, must be added into the former equation to account for the increase in taxes and therefore convenience, which we have done by multiplying the rate of recycling by this new factor. In theory, if there is a larger tax incentive presented, there will be a greater effort of the state to recycle and push recycling practices.

The factor t is defined as:

$$t = 1 + \frac{\text{incentive}}{90} \tag{21}$$

Where incentive is the percent of recycling costs given as a tax percentage. According to the EPA (29), the largest percentage of costs for the machinery used for recycling given back to the state as a tax incentive was 30% by Arkansas. However, as a result of this tax incentive, the rates of recycling for Arkansas only went up by 66% (30). As a result, the bottom of the fraction must be 90 in order for a 30% tax incentive to cause a 66% increase in the rate of recycling in a state.

While one would assume that the amount of trash would greatly decrease as a result of the tax incentive (or a price increase) and increasing rate of recycling, it would seem that this is not true. As shown by the city of Ann Arbor (25), which did attach a price per each can of garbage, there will not be a change in the garbage produced.

For the state of Utah, the state has a tax incentive of 20% recycling facility operating costs. For the state of Kansas, there currently is a tax incentive that is so small and so rarely used that it can be assumed to be 0 (31). The same has been found true of North Dakota (31).

By reevaluating the model for the tax incentive, the efficiency for Price, Utah (with the tax incentive) would be 0.000720313403. The efficiency for Wichita, Kansas and Fargo, North Dakota would remain the same as for regular single-stream recycling.

3.2.6 Results of the Second Model

From the results of the second model, it is immediately clear that adding on a tax incentive can only help to increase the efficiency of a population, in terms of recycling. The model, unfortunately, fails when tax incentives equal to 90% or greater of the costs of recycling are given back to a city. However, it is implausible that the costs of recycling would drop to 0 due to a 100% tax incentive. However, in ranges particularly close to current methods, efficiency goes up as the incentive grows. From the model, it is also clear that the tax incentive only creates a small increase. Unless the percent of costs that comes back as tax incentives rises to extraordinary levels, the tax incentive is not a singular factor that can single-handedly skyrocket efficiency rates. Of course, adding a tax incentive to less popular recycling methods may be effective because the tax incentive will directly affect the percent of the population that participates.

3.2.7 Rationale of the Third Model

The final model sought to model what would happen if a city had its residents pre-sort the different types of recyclable plastics and deliver to a specific location, which attempts to reduce the costs of transportation and sorting at the mill. In theory, there would have been a reduction in the efficiency due to the costs of transporting recyclables to the milling centers and sorting the goods. However, there is the inherent problem of your population. When a city implements such a program, it may discourage some folks from recycling. While they may have decided to recycle if they merely had to walk to the curb, the distance to the location dramatically increases in this model. Using many of the same principles as the previous two models, this model was constructed to see which factor was had a greater impact, whether the reduction in costs outweighed the drop in participation.

3.2.8 Pre-Sorted Recyclable Drop-Off Locations

The efficiency for individuals dropping off their pre-sorted recyclables at specific locations was modelled using the same general method as the previous two: Efficiency equals the total weight of the collected recyclables over the money spent to collect them:

$$E = \frac{N(\frac{r}{w \times x} \times m \times K \times L)}{N(2D + x \times d) + \frac{x \times C}{7} + x \times W \times 7 + x \times U - h \times Q \times N} \tag{22}$$

The new factors which appeared in the equation are: x, the number of divisions the container is separated into, U, the cost of the container, Q, the number of man hours used at the recycling center, and Q, the hourly wage at the aforementioned centers. The containers are separated by an average distance d, and their capacity is equal to the capacity of the waste disposal vehicles, 50 cubic yards; much was multiplied by a factor of x. The inconvenience is increased by a factor of x, since the recyclables must be sorted into x piles before they are placed into the containers. The distance between containers is increased by a factor of x, since the vehicles must move between them; the vehicles are only designed for single-stream containers, and as such can only handle on type of recyclable at a time. x number of vehicles, then, are needed to completely empty x number of containers. The cost of the container itself cannot be ignored, and the particular number of containers needed is x. The wages which are saved by having the recyclables presorted beforehand are also significant - they are subtracted out of the cost by multiplying total man-hours saved times the hourly wage of the workers. The types of materials which can be recycled are 7 types of plastic, paper, glass, and aluminum (27), bringing the value of x to 11. The average hourly wage of a plant worker is \$12.19 (28), and they work about 9 hours a day, with 20 workers in a plant (33). The values for the factors which are already known are assumed to be constant.

$$E = \frac{N(\frac{r}{w \times 11} \times m \times 0.0066978 \times 105)}{N \times (2 \times D + 11 \times d) + \$354,985.71 + 3073510 - 2194.2 \times N} \tag{23}$$

3.2.9 Results of the Third Model

From the results of the third model, it would seem that pre-sorted recycling at specific locations is the least efficient of the models. While it would save money at the mill because the recyclables have already been presorted, these costs are offset by the reduction in percentage of the population. The reduction of the population ready to recycle manifests as a result of the greater expenditure of effort needed by the consumers; though they may feel compelled to recycle more due to social pressures, the drawbacks of the design far outweigh its benefits, as is delineated through the final data obtained.

3.3 Analysis of the Models

In order to evaluate the accuracy and efficiency of the model, raw data was collected and evaluated into the functions explicated before.

3.3.1 Background on Cities

Before performing numerical analysis on the data, we first analyzed the cities under investigation. The city of Fargo, North Dakota has a population density of 2,162.0 per square mile according to the 2010 Census (32). As a result, this region of North Dakota is classified as an urban area and is the economic hub for eastern North Dakota. Much of the population, however, is involved with the medical and education industries occupations, comprising the top five occupations in the city, which gives the city an altruistic vibe. The area has a fairly developed recycling program, with dedicated weekly curbside pickup services and drop-off recycling programs.Yet the city does not have tax incentives that compel its residents to take part in the recycling program as depicted in the data obtained below for analysis purposes.

On the other hand, the city of Price, Utah has a density of 1,979.7 people per square mile (32). Thus, though its population is less than that of Fargo, due to its location in eastern Utah it is still categorized as an urban location. Its recycling program, unlike that implemented in Fargo, has a biweekly pickup system that runs every two weeks. Though its program is fairly developed, much of the population fails to participate, as the city has only a 6% recycling rate. However, with its recently implemented 20% tax incentive, participation in the city's recycling program has still risen considerably from the mere 3% of previous years citePrice. Thus, as indicated by our later analysis, this tax incentive has proven to be a significant encouraging factor in achieving greater recycling.

The city of Wichita, Kansas has a high population density akin to that of Fargo at 2,400 people per square mile (32). Wichita is the largest city in Kansas and comprises a large majority of the state's economy and population. Similar to that offered in Fargo, its city recycling program has both a weekly dedicated pick-up service and a drop-off program, and there are no tax incentives in place. Though its recycling rate of 25.3% (38) is considerably lower than the national average of 34.1 (37), it is still greater than those of the other cities under investigation.

3.3.2 Results

To complete analysis, we first needed to determine the values for the variables as per all the cities that were under investigation.

Each of the values in the table, with the exception of the population and rate of garbage visits, which were used to determine the values of the other variables, namely the total weight recycled and the number of garbage visits respectively, for the final efficiency calculation. Utah, as per its state legislation, has a recycling program that picks up recyclables once per

City	Efficiency (tons/$)
Fargo, ND	0.03257
Price, UT	0.0005893
Wichita, KS	0.2323

Table 2: Single-Stream Model without Tax Incentives

	Fargo, ND	Price, UT	Wichita, KS
Current Efficiency	0.03257	0.0007203	0.2323
Change in Efficiency (to 25%)	0.009048	0.00003274	0.02317
Percent Change	27.7778%	4.5455 %	27.7778 %
Efficiency with 25% Tax Incentive	0.04162	0.0007531	0.29689

Table 3: Results from the Single-Stream Model with Tax Incentives

two weeks, whereas each of the other cities has a program that picks recyclables weekly. The number of garbage truck visits, therefore, was 26, 52, and 52 for the cities respectively. After plugging in the data, the efficiencies for the cities were determined, shown in Table 2, 3, and 4.

3.3.3 National Average

The three recycling program models were then applied using national values in order to determine the prime candidate for a national recycling program. National average values were used for all variables to calculate the efficiency of the programs on a national scale. The results of the model are shown in Table 5 above.

3.4 Accuracy Analysis

In order to measure the accuracy of our models, we would compare them to historical data. Specifically, we would compare the models to towns which had recently switched to the system we were comparing. The analysis of the Township of South Orange above

City	Efficiency (tons/$)
Fargo, ND	0.0002943
Price, UT	0.000005161
Wichita, KS	0.002090

Table 4: Results from the Pre-Sorted Drop Off Location Model

Recycling Method	**Efficiency Achieved (tons/$)**
Single-Stream without Tax Incentives	494887.452
Single-Stream with Tax Incentives	632356.189
Pre-Sorted Drop Off Location Model	4320.350

Table 5: Results from the Pre-Sorted Drop Off Location Model

serves as a good example; the first town we would compare would be the town of Dedham, Massachusetts, which fits all of the above criteria.

3.5 Sensitivity Analysis

In order to measure the sensitivity of our equations and the variable that contributes the most to change, we used differentials on each of our equations.

3.5.1 Single-Stream Recycling Analysis

The total differential of the equation for efficiency is equal to:

$$dE = \frac{\partial E}{\partial D} \cdot dD + \frac{\partial E}{\partial d} \cdot dd + \frac{\partial E}{\partial m} \cdot dm + \frac{\partial E}{\partial r} \cdot dr + \frac{\partial E}{\partial w} \cdot dw + \frac{\partial E}{\partial N} \cdot dN + \frac{\partial E}{\partial p} \cdot dp \tag{24}$$

Each term in the equation of the differential represents the change in the final efficiency with respect to a change in one of the variables. The partial differential of the efficiency with respect to a variable shows how much the final efficiency changes with respect to only the variable specified. By multiplying that partial differential by the change in the variable, we can estimate the change due to a specific parameter. Summed together, this total differential represents the complete effect of changes in each of the variables on the output of the function.

The total differential is evaluated with the current values of each of the variables. In addition, the differential terms (of the form dA, where A is any of the variables) will be set equal to one percent of the original value. This produces a total differential which calculates the effect of shifting all variables 1%. To determine which of the variables has the greatest effect on the output of the function, each term of the total differential is represented as a percentage of the total differential, representing the relative effect of that term on the function. The term with the highest percentage thus has the greatest effect, and the function is most sensitive to this term or variable.

The sensitivity for the other two equations would be performed in the same manner as that described above.

4 Recommendations and Conclusions

From the models generated and the results obtained from the data, we recommend that more cities in the United States implement single-stream recycling with tax incentives. Not only would this afford the recycler the convenience associated with reduced sorting effort, but collection costs would also decrease significantly. With respect to the municipal effects of instituting this policy, recycling collection trucks would have greater fleet flexibility due to the lack of need for specialized trucks, and a greater variety of waste products could be recycled.

Furthermore, recycling plastic is a positive externality and is likely consumed at a rate less than the socially optimal quantity. As such, providing a tax incentive, a form of government subsidy, would increase the marginal benefit and compel more citizens and businesses to recycle. The costs of separating the recycled materials would be compensated for by an increase in carbon credits (due to the decrease in carbon emissions), the reduced need to buy more land to dispose waste in, and the profits generated by the recycled materials. Fuel consumption all across the nation would decrease because less vehicles would be needed to transport the waste to landfills, and less environmental harm would be inflicted from the accumulation of plastic waste.

Since some states profit from charging other states for disposing municipal waste on their land, we also recommend that the federal government impose fines on states that transport their waste across state lines. Considering that certain states, such as Massachusetts and Rhode Island, have less than 12 years of landfill capacity left, these states will have greater motivation to recycle (40). In addition, business-sponsored recycling incentives, such as Earth Aid, a business that raised $4 million in 2011 from rewarding citizens for consuming less energy and water, have also been popular in the public eye. It would behoove the federal government to provide more grants to such businesses because it promotes environmentally friendly practices throughout the US without the need to create additional or modify existing infrastructure.

In conclusion, our mathematical models have demonstrated that plastic waste production is rapidly approaching unsustainable levels in the United States and that increasing plastic recycling is critical towards the nation's future well-being. To this end, we have analyzed three major types of plastic waste recycling and concluded that single-stream recycling in combination with the institution of tax incentives is the optimal method for doing so. Although the United States still has a long way to go, these are the first steps we can take on the road to a sustainable future.

References

[1] "Autocar Pendpac Side Loader Garbage Truck For Sale — TW1120199SL." *Autocar Pendpac Side Loader Garbage Truck For Sale — TW1120199SL.* Autocar, n.d. Web. 03 Mar. 2013.

[2] "Bid Specifications for Town of Huntsville Residential Garbage Collection." Municipal Technical Advisory Society. The University of Tennessee, n.d. Web. 3 Mar. 2013.

[3] Brooks, David W. "Uses of Plastics." *Plastics Resource.* Natural Science Foundation, n.d. Web. 03 Mar. 2013.

[4] "Environmental Activism." *International Communities Website.* Fellowship for International Communities, n.d. Web. 03 Mar. 2013.

[5] Minnesota. Minnesota Pollution Control Agency. Tim Goodman and Associates. *SINGLE-STREAM AND DUAL-STREAM RECYCLING: COMPARATIVE IMPACTS OF COMMINGLED RECYCLABLES PROCESSING.* By Tim Goodman. St. Paul: n.p., 2006. Print.

[6] "Nylon Is Invented." *PBS.* PBS, n.d. Web. 03 Mar. 2013.

[7] "Thermoplastic Resin Sales by Major Market 2007-2011." *American Chemistry Council.* Veris Consulting, Inc., 2012. Web. 3 Mar. 2013.

[8] "Time It Takes for Garbage to Decompose in the Environment:." *New Hampshire Department of Environmental Services.* U.S. National Park Service, n.d. Web. 3 Mar. 2013.

[9] *Plastic Recycling Collection: National Reach Study.* Rep. Moore Recycling Associates Inc., May 2011. Web. 3 Mar. 2013.

[10] "Shipping Methods." *96 Gal. Wheeled Trash Can Cart-025596-01GRS* The Home Depot, n.d. Web. 03 Mar. 2013.

[11] United States of America. Bureau of Labor Statistics. Occupational Employment and Wages. *U.S. Bureau of Labor Statistics.* U.S. Bureau of Labor Statistics, n.d. Web. 03 Mar. 2013.

[12] United States of America. Environmental Protection Agency. Full Cost Accounting. *Allocating Costs.* N.p.: n.p., n.d. Environmental Protection Agency. Web. 3 Mar. 2013.

[13] United States of America. Environmental Protection Agency. Resource Conservation and Recovery. *Municipal Solid Waste Generation, Recycling, and Disposal in the United States Tables and Figures for 2010.* N.p.: n.p., n.d. United States Environmental Protection Agency. Web. 03 Mar. 2013.

[14] Rust, Dwight A. "The Thermoset Resin Industry - A New Business Model?" *Composites and Polycon* (2009): 1-4. Print.

[15] Sidique, Shaufique Fahmi. *ANALYSES OF RECYCLING BEHAVIOR, RECYCLING DEMAND, AND EFFECTIVENESS OF POLICIES PROMOTING RECYCLING.* Diss. Michigan State University, 2008. N.p.: n.p., n.d. 2008. Web. 3 Mar. 2013.

[16] McKee, Fred. *Truck Engine Round Table.* Jackson Hole, Wyoming: Mack Trucks, Inc., n.d. PPT.

[17] "Refuse." *South Orange Village Division of Public Works :: Maintenance.* South Orange Village, n.d. Web. 03 Mar. 2013.

[18] "Recycling." *South Orange Village Division of Public Works :: Maintenance.* South Orange Village, n.d. Web. 03 Mar. 2013.

[19] "South Orange Crime Statistics and Crime Data (South Orange, NJ)." *South Orange*

Crime Statistics and Crime Data N.p., n.d. Web. 03. Mar. 2013.

[20] "South Orange Recycling." *The Township of South Orange Village.* South Orange Village, n.d. Web. 3 Mar. 2013.

[21] "Single Stream Recycling Increases Recycling Rates." *Single Stream Recycling Increases Recycling Rates*. Greenopolis, n.d. Web. 03 Mar. 2013.

[22] Maier, Karyn. "How Much Does One Household Produce in Recyclables?" *Green Living on National Geographic.* N.p.n.d. Web. 03 Mar. 2013.

[23] DeLeon, G., P. L. Neidert, B. M. Anders, and V. Rodriguez-Catter. "Choices between Positive and Negative Reinforcement during Treatment for Escape-maintained Behavior." *Journal of Applied Behavior* 34.4 (2001): 521-25. US National Library of Medicine. Web. 3 Mar. 2013.

[24] "Plastics' Contribution to the US Economy." Economic Statistics. *Spi: the Plastics Industry Trade Association*, n.d. Web. 03 Mar. 2013.

[25] Askins, Dave. "Recycling, Yes for Now; Public Art, Postponed." *The Ann Arbor Chronicle.* The Ann Arbor Chronicle, 22 Sept. 2011. Web. 03 Mar. 2013.

[26] Knoblauch, Jessica A. "Plastic Not-So-Fantastic: How the Versatile Material Harms the Environment and Human Health." *Scientific American*. N.p., 2 July 2009. Web. 03 Mar. 2013.

[27] "RecyclingCenters.org." *Local Recycling Centers and Recycling Information and Statistics. Learn How to Recycle and Live Green* -. Recycling Centers, n.d. Web. 03 Mar. 2013.

[28] "Recycle Worker." *Salary and Job Description*. Kaplan University, n.d. Web. 03 Mar. 2013.

[29] "State Recycling Tax Incentives." *EPA*. Environmental Protection Agency, n.d. Web. 03 Mar. 2013.

[30] Sadler, Aaron. "TAX CREDIT PROVIDES RECYCLING INCENTIVE." *ADEQ*. Arkansas Department of Environmental Quality, 28 Jan. 2010. Web. 3 Mar. 2013.

[31] Sparks, Kathern. "Tax Credits: An Incentive for Recycling?" *EPA*. Enhvironmental Protection Agency, July 1998. Web. 3 Mar. 2013.

[32] U.S. Department of Commerce. "2010 Census of Population and Housing." *2010 Census* 1.3 (2012): 1-730. Print.

[33] "Recycle Worker." *Salary and Job Description.* N.p., n.d. Web. 03 Mar. 2013.

[34] "Fargo Recycles." *City of Fargo.* N.p., n.d. Web. 03 Mar. 2013.

[35] "The Recycling Programs." *Price City, Utah.* N.p., n.d. Web. 03 Mar. 2013.

[36] "LEED Construction Waste Management: Green Recycling Methods for Reducing Carbon Footprint by JR Riddle." *Urban One RSS.* N.p., n.d. Web. 03 Mar. 2013.

[37] "Municipal Solid Waste (MSW) in the United States: Facts and Figures." *US Environmental Protection Agency*, 15 Nov. 2012. Web. 3 Mar. 2013.

[38] "Acceptable Items." *Recycling in Wichita Kansas.* N.p., n.d. Web. 03 Mar. 2013.

[39] "SPI Resin Identification Code - Guide to Correct Use." *SPI.* The Plastics Industry Trade Association, n.d. Web. 03 Mar. 2013.

[40] Palmer, Brian. "Go West, Garbage Can!" *Slate Magazine*. The Slate Group, 15 Feb. 2011. Web. 03 Mar. 2013.